A SEQUENTIAL MONTE CARLO APPROACH TO COMPUTING TAIL PROBABILITIES IN STOCHASTIC MODELS by Hock Peng Chan and Tze Leung Lai

PI website: www.stat.nus.edu.sg/~hockpeng

The past decade has witnessed many important advances in Monte Carlo methods for computing tail distributions and boundary crossing probabilities of multivariate random walks with independent and identically distributed (i.i.d.) or Markov-dependent increments. In particular, the case of heavy-tailed random walks has attracted much recent attention because of its applications to queueing and communication networks. Another area of much recent interest is the development and the associated probability theory of efficient Monte Carlo methods to compute rare-event probabilities.

Professor Chan (and his collaborator, Professor T.L. Lai of Stanford University) have proposed a new approach to simulating rare-event probabilities for heavy-tailed random walks. Their approach uses not only sequential (dynamic) importance sampling but also resampling. An important idea underlying the method to simulate rare-event probabilities for heavy-tailed random walks is to make use of the single large jump property to decompose the event of interest into two disjoint events, one of which involves the maximum increment being large. Different Monte Carlo schemes are then used to simulate these two events.

In complex stochastic models, it is often difficult to evaluate probabilities of events of interest analytically and Monte Carlo methods provide a practical alternative. When an event occurs with a small probability, a very large number of runs need to be generated for direct Monte Carlo computation to work. Instead of direct Monte Carlo computation, one can make use of importance sampling by changing the measure to one under which the event is no longer a rare event and evaluating the desired (rare-event) probability by weighting the estimate obtained under the new measure with an appropriate likelihood ratio. Unfortunately, it has been demonstrated that importance sampling measures that are consistent with large deviations can perform much worse than direct Monte Carlo computation. This problem can be resolved by using certain mixtures of exponentially twisted measures that are asymptotically optimal for importance sampling. For complex stochastic models, however, there are difficulties with implementation in using these asymptotically optimal importance sampling measures. Chan and Lai have introduced a sequential importance sampling and resampling (SISR) procedure to attain a weaker form of asymptotic optimality. The objective is to approximate the target measure by the weighted empirical measure defined by the resampling weights. The SISR procedure to compute probabilities of rare events is closely related to (a) the interacting particle systems (IPS) approach introduced by Del Moral and (b) the dynamic importance sampling method introduced by Dupuis and Wang.