

Mathematical Models and Numerical Methods for Dipolar Bose-Einstein Condensates

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Since the first observation of a Bose-Einstein condensate (BEC) in 1995, quantum degenerate gases have attracted a great deal of theoretical and experimental interest. These dilute atomic gases forming at near absolute zero temperatures provide opportunities for studying ultracold atomic systems, superfluids and quantum turbulence, enhancing research activities related to low temperature physics, bright atomic lasers, nonlinear (quantum) optics and quantum transport.

The properties of BECs are normally determined by the short-range, isotropic contact interactions between the particles. Yet interest in the area intensified recently with the successful experimental realisation of BECs with dipole-dipole interactions – dipolar BECs.

Prof Weizhu Bao, who leads a team researching dipolar BECs, explains that they are particularly important because “the long-range and anisotropic nature of the dipolar interaction greatly enriches the static and dynamic properties of ultracold degenerate quantum gases”. However, these BECs are very difficult to model. Over the past three years, the team has been able to address that situation, developing new mathematical models, establishing rigorous theories and designing the most efficient and accurate numerical methods for computing the ground states and dynamics of dipolar BECs.

While conventional methods of modelling dipolar BECs are limited to “a few specific dipolar orientations”, these new methods allow modelling in one or two dimensions “under arbitrary dipolar orientation”. Prof Bao’s team found that by decoupling dipole-dipole interaction potential into short-range (local) and long-range (repulsive and attractive) interactions, the Gross-Pitaevskii equation conventionally used to model BECs is reformulated as a Gross-Pitaevskii-Poisson type system. Based on this new formulation, they derived rigorous new one- and two-dimensional mathematical models for cigar- and pancake-shaped dipolar BECs with arbitrary polarisation angles.

The team also proved the existence, uniqueness, and the nonexistence, of the ground states in three dimensions, two dimensions and one dimension, and established the existence of a global weak solution and finite time blowup of the dynamics under various parameters. Their work produced numerical methods that, as Prof Bao puts it, “improve computational accuracy by several orders of magnitude from previous methods”

The findings can be used by physicists and mathematicians in work related to dipolar BECs covering such areas as superfluid properties, soliton-like solutions, dipolar tunnelling, Anderson localisation under random potential and spin-orbital coupling. Prof Bao notes that his own team is looking to focus on “quantized vortex and superfluid properties related to rotating dipolar BECs under arbitrary dipole orientation, spinor dipolar BECs and degenerate dipolar Fermion gas, as well as dipolar BECs under finite temperature”. They also intend to work in areas such as solid-state dewetting problems that have applications in materials science and multiscale methods and analysis using Dirac equations with applications in graphene research.

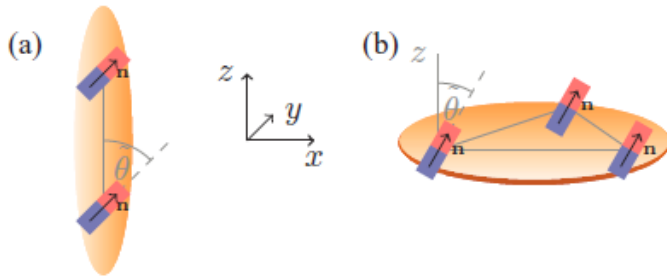


Fig 1. In the quasi-1D setup in (a) the dipolar BEC is confined to the z direction, and in the quasi-2D setup in (b) the dipoles are confined to the x - y plane [2].

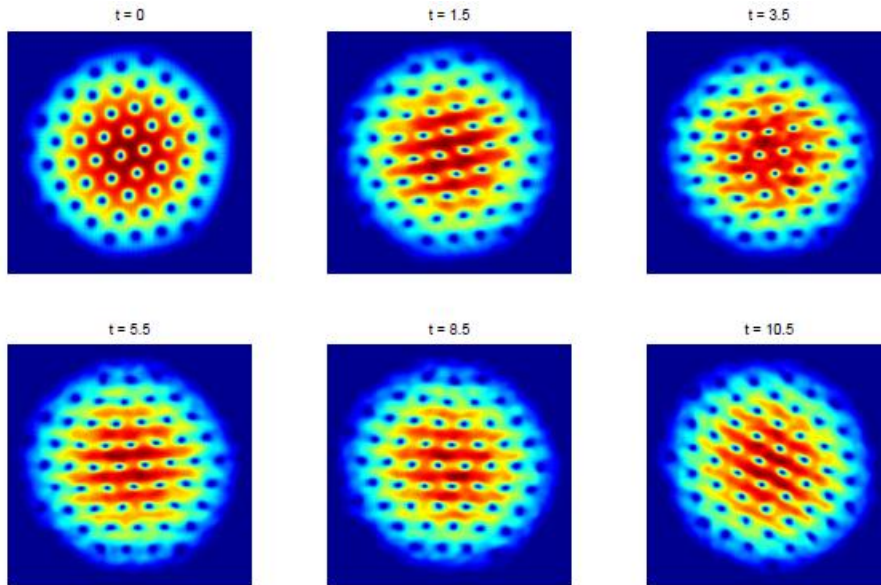


Fig 2. Dynamics of a quantised vortex lattice in a rotating dipolar BEC [5].

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