Long-Lasting Exponential Spreading in Periodically Driven Quantum Systems:

The rate of quantum wavepacket spreading is an old problem but not well understood. Well-known examples include ballistic diffusion, normal diffusion, and anomalous diffusion. Quite recently, even quantum hyper-diffusion, namely, wavepacket spreading faster than ballistic diffusion, has also been carefully studied in a simple setup [1]. But is it possible to have exponential wavepacket spreading in nontrivial quantum systems?

One often associates exponential behavior with classical chaos, namely, exponential sensitivity to initial conditions. However, due to the complicated stretching and folding dynamics in classical phase space, exponential sensitivity itself does not directly give rise to exponential growth in physical observables. Noticing also that in general true chaos does not exist in quantum dynamics, it seems very rare to have long-lasting exponential spreading in quantum systems. Nevertheless, in a recent publication, A/P Gong Jiangbin and his collaborators from China and Italy have shown, for the first time, how longlasting exponential growth of momentum variance (or energy) may occur in a simple dynamical model whose classical limit is chaotic [2]. As an original component in the study, a driven quantum system in its deep quantum regime is tuned close to a certain type of quantum resonance, such that the dynamics of the close-to-resonance quantum system behaves extremely close to a pseudo-classical quasi-integrable system with an unstable fixed point. Furthermore, the driving fields were designed such that the initial quantum state can be easily placed astride the stable manifold of the fixed point. As shown in the study, the ensuing quantum evolution then resembles to the repelling classical dynamics around the fixed point, thereby establishing long-lasting exponential behavior without physically tuning the quantum system to its semiclassical limit. In one of the computational examples discussed in the publication [2] (also see figure on the next page), an exponential increase in momentum variance covering about 7 orders of magnitude can be obtained, before some sign of saturation sets in.

Because exponential spreading is fundamentally different from all previously known cases of wavepacket dynamics with polynomial time dependence, it represents a novel quantum dynamical phenomenon and is hoped to be a new element in the so-called quantum chaos studies. Exponential wavepacket spreading dynamics might be useful for designing some robust acceleration methods. The finding of long-lasting exponential wavepacket spreading should further motivate the use of near quantum-resonance dynamics to explore fascinating phenomena in quantum evolution.

References:

- [1] Z.J. Zhang, P.Q. Tong, J.B. Gong, and B.W. Li, **Phys. Rev. Lett**. Vol 108, 070603 (2012), Quantum Hyperdiffusion in One-dimensional Tight-Binding lattices.
- [2] J. Wang, I. Guarneri, G. Casati, and J.B. Gong, **Phys. Rev. Lett.** Vol. 107, 234104 (2011), Long-Lasting Exponential Spreading in Periodically Driven Quantum Systems.



Figure taken from Ref. [2], depicting two examples of the time dependence of the momentum variance (logarithmic scale) in a driven quantum system. The dashed fitting straight lines indicate that the shown time dependence is exponential over a long time scale. In panel (a), the exponential spreading covers a range of momentum variance around 7 orders of magnitude. The dotted line in panel (a) represents the results obtained from a pseudo-classical system, demonstrating that a near-resonance quantum system in its deep quantum regime might be well described by classical dynamics.