Localized Realized Volatility Modeling[1]

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Volatility is one of the key elements in modeling the stochastic dynamic behavior of financial assets. It is not only a measure of uncertainty about returns but also an important input parameter in derivative pricing, hedging and portfolio selection. Accurate volatility modeling is therefore in the focus of financial econometrics and quantitative finance research. One of the stylized features of volatility is the long range dependence. This feature is illustrated in Figure 1, which depicts the daily sample autocorrelation functions of daily logarithmic realized volatility of the S&P500 index futures for a long sample period (1985-2005) and for a short sample period (1995). The sample autocorrelation function has typically a hyperbolically-like decaying shape, also known as "long memory".

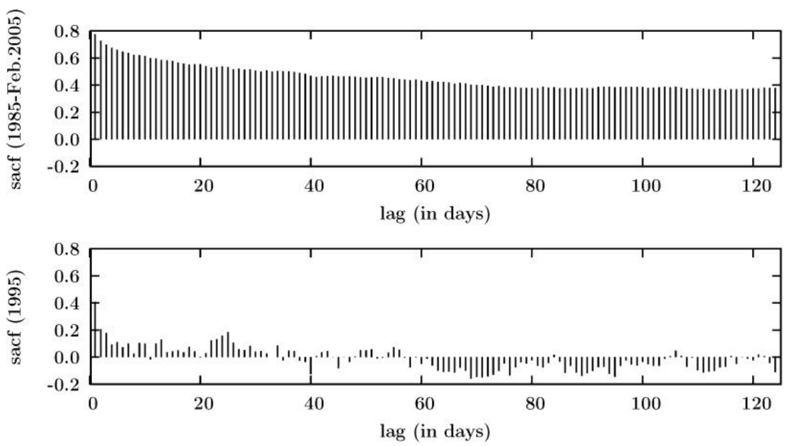

Figure 1: Sample ACF plots of daily logarithmic realized volatility of the S&P500 index futures for the sample from 1985-2005 (upper panel) and for the year 1995 (lower panel).

With the recent availability of high-frequency financial data the long range dependence of volatility regained researchers' interest and has led to the consideration of long-memory models for volatility such as HAR (Corsi 2009), ARFIMA (Andersen et al. 2003), A-ARFIMA (Baillie and Morana 2009) models. The long range diagnosis of volatility, however, is usually stated for long sample periods, while for small sample sizes, such as e.g. one year, the volatility dynamics appears to be better described by short-memory processes. The ensemble of these seemingly contradictory phenomena point towards short-memory models of volatility with nonstationarities, such as structural breaks or regime switches, that spuriously generate a long memory pattern.

In this paper we adopt this view on the dependence structure of volatility and propose a localized Autoregressive (LAR) procedure for modeling the logarithmic realized volatility that is often Gaussian distributed:

$$\log RV_t = \theta_{0t} + \theta_{1t} \log RV_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_t^2)$$

That is at each point in time we determine a past interval over which the log volatility is approximated by a local linear process. A simulation study shows that long memory processes as well as short memory processes with structural breaks can be well approximated by this local approach. Furthermore, using S&P500 data we find that our local modeling approach outperforms long memory type models and models with structural breaks in terms of predictability. Table 1 presents the RMSFEs of the LAR model and the alternative models for

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the 1-, 5- and 10-day ahead forecasts. The empirical results reveal that the adaptive approach introduces more flexibility into the procedure and results in an increase in forecast accuracy.

Table 1.Root mean square forecast errors and information sets of the best models

	1-day ahead forecast		**5-day ahead forecast**		**10-day ahead forecast**	
Model	RMSFE	Info set	RMSFE	Info set	RMSFE	Info set
LAR	**0.4791**	6m	**0.4619**	1m	**0.4615**	1m
AR	0.5047	3m	0.5712	3m	0.5873	3m
STR-Tree	0.5547	rec.	0.7746	rec.	0.8738	rec.
ARFIMA	0.4991	3y	0.5827	3y	0.6207	3y
A-ARFIMA	0.5020	4.5y	0.5904	4.5y	0.6312	4y
HAR	0.5014	3y	0.5848	2.5y	0.6232	2.5y

The table reports the root mean square forecast errors (RMSFE) of the h-day ahead logarithmic realized volatility forecasts of the S&P500 index futures based on the various models. “Info set" refers to the corresponding sample size used in the computation of the critical values (for the LAR procedure) or to the size of the rolling window used in model estimation and prediction (for the AR, ARFIMA and HAR models). “rec." refers to forecasts based on the STR-Tree model (Scharth and Medeiros 2009), for which the recursive forecasting scheme is employed.

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