NATIONAL UNIVERSITY OF SINGAPORE

Advanced Placement Test

PC1144 INTRODUCTION TO MODERN PHYSICS

(Sample Paper)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. This assessment paper contains **FOUR short** questions in Part I and **THREE long** question in Part II. It comprises **5** printed pages.
2. Students are required to answer **ALL** questions.
3. Answers to the questions are to be written in the answer booklet.
4. This is a CLOSED BOOK examination.
5. Only non-programmable and non-graphing calculators without remote communication function may be used.
6. Total marks for Part I is 40 and that for Part II is 60.

Part I: Short Questions

1. 10^6 muons are created 10 km above the surface of Earth. Muons have a mean lifetime of 2.2 μs. Suppose the muons are travelling with speed 0.98 c.

   (a) From the muon's reference frame, find the time needed for them to reach the surface of Earth. [5 marks]

   (b) Determine the number of muons reaching the surface of Earth. [5 marks]

2. A monochromatic beam of light is absorbed by a collection of ground state hydrogen atoms. It is observed that there are ten emissions of different wavelengths when the hydrogen de-excites back to the ground state. What is the wavelength of the incident beam of light?

   [The energy levels of the hydrogen atom is $E_n = -\dfrac{13.6}{n^2}$ eV, where n = 1, 2, $\cdots$.] [10 marks]

3.
   (a) Describe briefly the Liquid drop model and the Shell model. [6 marks]

   (b) Using the decay law $N(t) = N_0 \exp(-\lambda t)$, the change in the number of nuclei ΔN between t and $t + \Delta t$ is approximated to be $|\Delta N| = N(t) - N(t + \Delta t) \approx k N_0 \Delta t \exp(-\lambda t)$ where $\Delta t << 1/\lambda$ and k is an unknown. Find k. [4 marks]

4. The following statements are INCORRECT. Explain why.

   (a) The electronic charge of mesons ranges from – 2e to + 2e. [3 marks]

   (b) The reaction $p + p \to p + p + p + \Omega^-$ can take place. [3 marks]

   (c) An electron with momentum 5 MeV/c has kinetic energy of about K = 24.5 MeV. [Hint: Mass of electron = 0.511 MeV/c^2] [4 marks]

Part II: Long Questions

5. If a photon with energy E is scattered by a free electron (initially at rest) through an angle θ, the scattered photon will have energy E'.

   (a) From the principles of conservation of energy and conservation of momentum, show that the scattered photon's energy is given by

$$E' = \frac{E}{1 + \frac{E}{mc^2}(1-\cos\theta)},$$

   where m is the rest mass of the electron. [10 marks]

   (b) The kinetic energy gained by the electron can be expressed as

$$T_e = \frac{E^2(1-\cos\theta)}{kmc^2 + E(1-\cos\theta)},$$

   where k is a constant. Find the unknown k. [4 marks]

   (c) Determine the maximum kinetic energy gained by the electron. What is the scattering angle for this case? [6 marks]

6. A set of wavefunctions for a particle with mass m in an infinite square well of width L is given by

$$\Psi_n(x,t) = \begin{cases} \sqrt{\dfrac{2}{L}} \sin\left(\dfrac{n\pi}{L} x\right) e^{-iE_n t/\hbar} & , 0 \le x \le L, \\ 0 & , \text{elsewhere}, \end{cases}$$

where n = 1, 2, 3, $\cdots$ and $E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$. Suppose that a particle is described by a normalised wavefunction

$$\Phi(x,t) = \frac{1}{\sqrt{3}}\Psi_1(x,t) + \frac{i}{\sqrt{6}}\Psi_3(x,t) + q\Psi_5(x,t),$$

where q is a real positive constant.

(a) Find the unknown q. $\left[\text{Hint} : 2\sin A \sin B = \cos(A-B) - \cos(A+B)\right]$ [8 marks]

(b) Is the wavefunction $\Phi(x, t)$ stationary? Give reasons for your answer. [6 marks]

(c) What are the probabilities of finding the particle with energy E_1, E_3 and E_5 respectively? [6 marks]

7. In an experiment, 15 keV electrons were fired at a block of brass (copper-zinc alloy).

(a) Describe what would happen when the electron hit the metal. [2 marks]

(b) Sketch or describe the spectrum of radiation produced from this experiment. [3 marks]

The K_α lines of an element are produced when an electron from K shell is knocked out and another electron from the L shell fills its place. These lines have transition frequencies that are approximately given by Moseley's law: $f = \left(2.48\times10^{15}\text{ Hz}\right)\left(Z-1\right)^2$ where Z is the proton number of the atom.

(c) Describe how the K_α lines will affect the spectrum of radiation from this experiment. (Copper: Z = 29 , Zinc: Z = 30) [5 marks]

(d) Using Moseley's law, suggest how we can identify the material in an unknown sample using 15 keV electron beam. [3 marks]

(e) Using the 15 keV electron beam, can we produce K_α lines from silver (Z = 47) atoms? Explain your reasoning. [4 marks]

(f) Other transitions may also take place. Would K_β line (M shell to K shell) have a higher or lower energy than the K_α line? Explain why. [3 marks]

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