NATIONAL UNIVERSITY OF SINGAPORE

Advanced Placement Test

PC1143 Introduction to Electricity & Magnetism

(Sample Paper)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **5 short** questions in Part I and **3 long** questions in Part II. It comprises **9** printed pages.
2. Answer **ALL** the questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. The total marks for Part I is 40 and that for Part II is 60.
6. Only non-programmable and non-graphing calculators without remote communication function may be used.
7. A table of constants and mathematical formulae is attached.

PART I

This part of the examination paper contains **five** short-answer questions on pages 2 to 4. Answer **ALL** questions.

Question 1

Consider the box-like Gaussian surface *G* as shown in Figure 1.

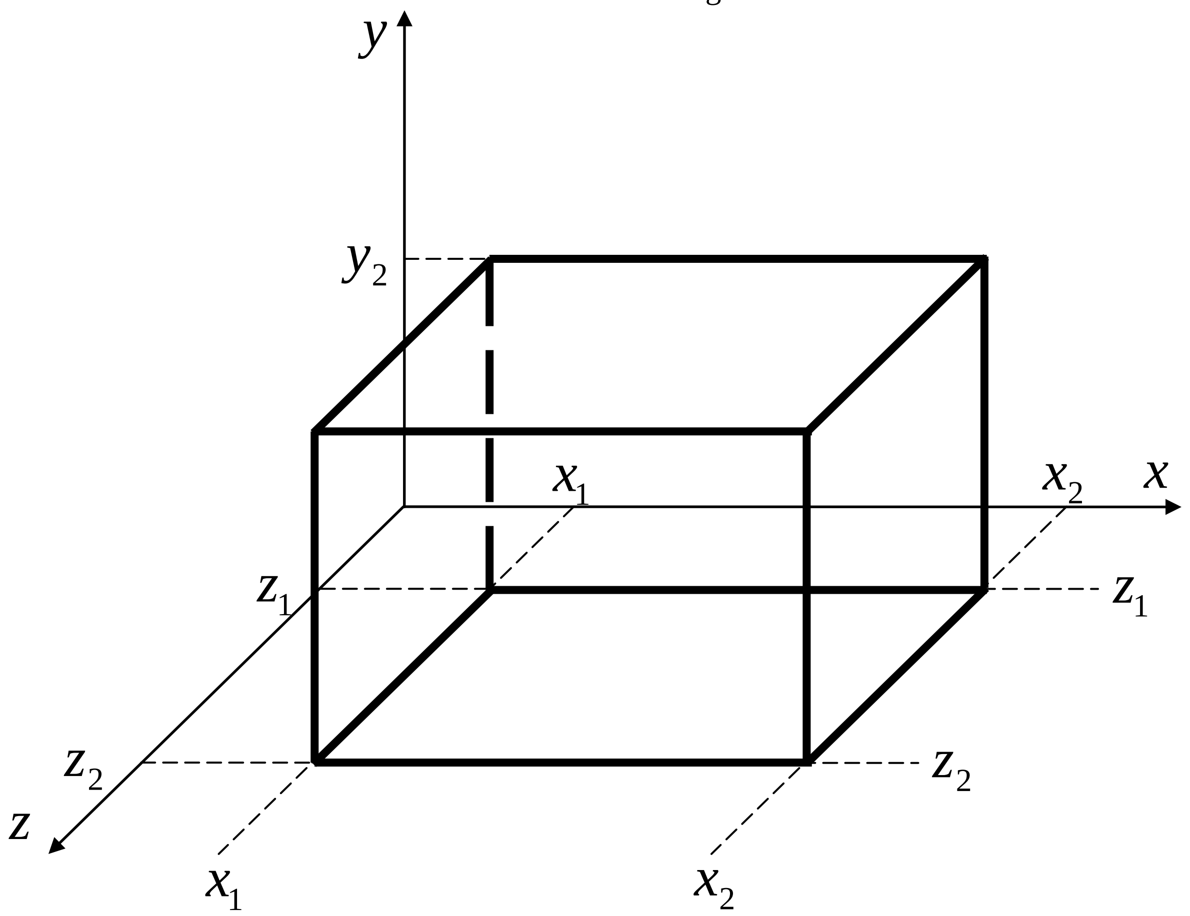

Figure 1

The bottom face is in the *xz*-plane, while the top face is in the horizontal plane passing through $y_2 = 2.00\ \text{m}$. Suppose $x_1 = 1.00\ \text{m}$, $x_2 = 4.00\ \text{m}$, $z_1 = 1.00\ \text{m}$, and $z_2 = 3.00\ \text{m}$, and *G* lies in an electric field given by

$$\vec{E} = [(-6.00x + 10.0)\hat{i} - 3.00\hat{j} + \kappa z\hat{k}]\ \text{N/C},$$

with *x* and *z* in metres and κ is a constant. If *G* encloses a net charge of $-48.0\varepsilon_0$ C, find κ. [8]

Question 2

(a) Consider a particle *P* of mass *m* and positive charge *q* moving with velocity $\vec{v}$ in a uniform static magnetic field $\vec{B}$.

(i) Describe the magnitude and direction of the magnetic force that $\vec{B}$ exerts on *P*. [3]

(ii) Can $\vec{B}$ do work on *P*? Explain briefly. [2]

(b) A source injects an electron of speed $v = 1.5\times10^6$ m/s into a uniform magnetic field of magnitude $B = 1.0\times10^{-2}$ T. The velocity of the electron makes an angle $\theta = 30^\circ$ with the direction of the magnetic field. Find the distance *d* from the point of injection at which the electron next crosses the field line that passes through the injection point. [3]

Question 3
Figures 2 and 3 show two circuits, each with $R = 14.0\ \Omega$, $C = 6.20\ \mu F$, $L = 56.0$ mH, and a battery having emf $\mathsf{E} = 34.0$ V.

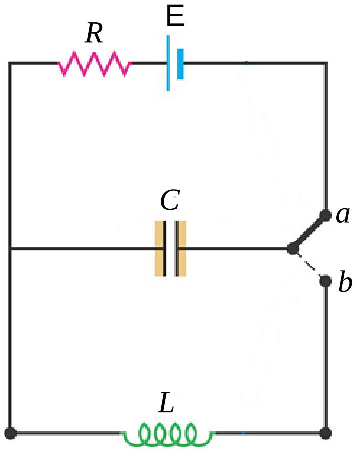

Figure 2

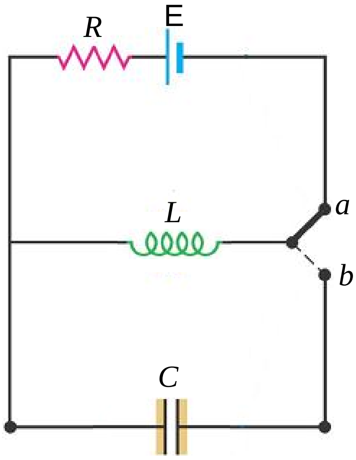

Figure 3

The switch in Figure 2 is kept in position *a* for $2\tau_C$ s, while the switch in Figure 3 is kept in position *a* for $2\tau_L$ s. The switches are then separately thrown to their respective position *b*'s. Here, τ_C and τ_L are the *capacitive* and *inductive time constants* respectively. Find

(a) τ_C and τ_L. [2]

(b) the angular frequency of the resulting oscillations in each circuit. [1]

(c) the current in each circuit 1.20 ms after the switches are being thrown to their respective position *b*'s. [5]

Question 4
A series *R*-*L*-*C* circuit (Figure 4) consists of a resistor $R = 100\ \Omega$, an inductor $L = 0.15$ H, and a capacitor $C = 30\ \mu F$ connected in series to an 120 V (rms) emf source. The frequency of the source is 60 Hz.

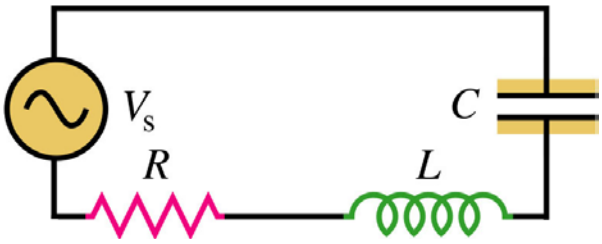

Figure 4

Calculate

(a) the current amplitude. [4]

(b) the phase angle ϕ. [2]

(c) the average power loss. [2]

Question 5

A plane electromagnetic wave travelling in the positive direction of an *x*-axis in vacuum has electric field components $E_x = E_y = 0$ and

$$E_z = E_{\max} \cos(\omega t - kx),$$

with $E_{\max} = 2.0\ \mathrm{V/m}$ and $\omega = \pi \times 10^{15}\ \mathrm{s}^{-1}$.

(a) Find *k*. [1]

(b) What is the magnetic field associated with the wave? [3]

(c) Calculate the wave intensity. [2]

(d) The wave uniformly illuminates a surface of area $2.0\ \mathrm{m}^2$. If the surface totally absorbs the wave, calculate the radiation pressure and the magnitude of the corresponding force on the surface. [2]

END OF PART I

PART II

This part of the examination paper contains **three** long questions on pages 5 to 9. Answer **ALL** questions.

Question 6(a)

Positive electric charge Q is uniformly distributed along a non-conducting rod R with length L, lying along the x-axis between $x = 0$ and $x = L$, as shown in Figure 5.

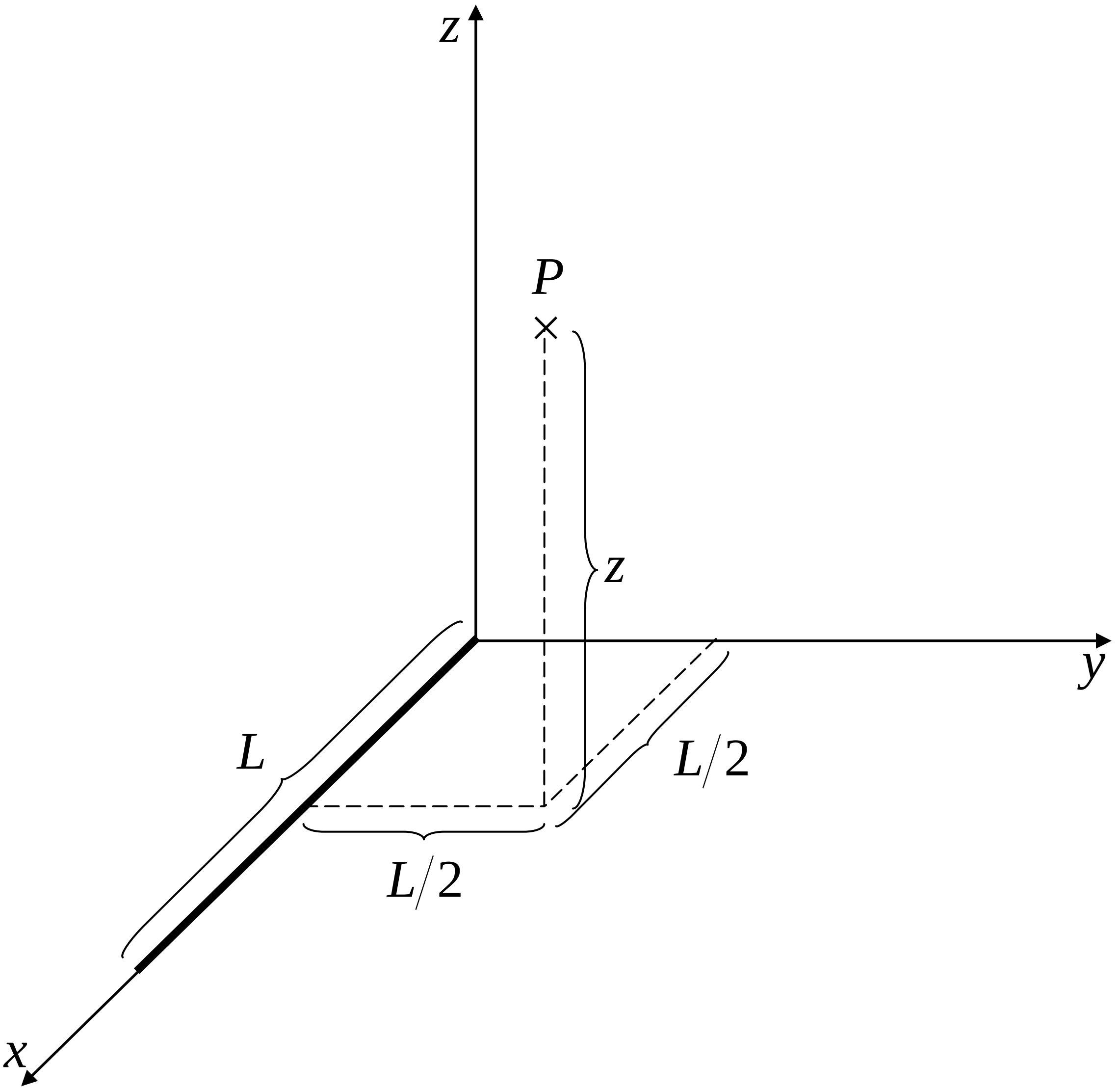

Figure 5

Consider a point P that lies on a line, which goes through a point on the xy-plane with coordinates $x = L/2$ and $y = L/2$, and is parallel to the z-axis. P is at a height z above the xy-plane. Show that the electric potential V at P is given by

$$V(z) = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{\sqrt{2L^2 + 4z^2} + L}{\sqrt{2L^2 + 4z^2} - L}\right).$$

[7]

Question 6(b)
Now, we have four such rods as R, that are arranged into a square as shown in Figure 6.

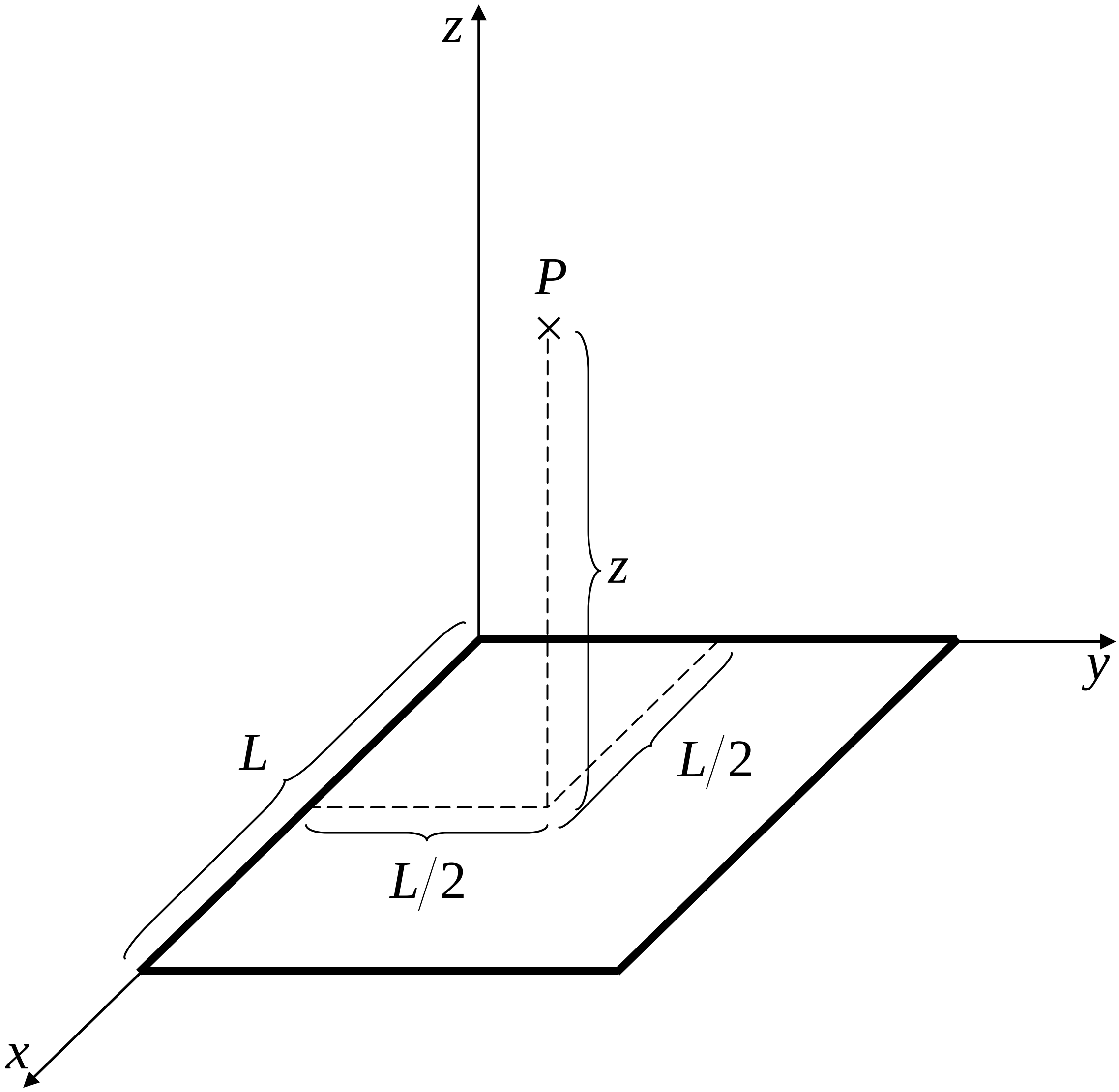

Figure 6

Find the electric field $\vec{E}$ at point P. Explain clearly how you obtain your answer. [9]

Question 6(c)
If an electron of mass m is placed at the centre of the square and is then displaced a small distance z along the vertical dashed line as shown in Figure 6 $(z \ll L)$, determine its oscillating frequency. You may ignore gravity. [4]

Question 7(a)
Consider a positive point charge q moving with velocity $\vec{v}$ as shown in Figure 7.

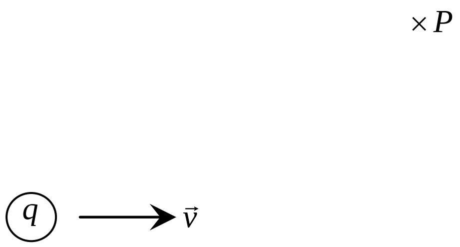

Figure 7

Describe the magnitude and direction of the magnetic field at point P due to q. [6]

Question 7(b)
Hence, derive the infinitesimal magnetic field $d\vec{B}$ at point P due to a current element $Id\vec{l}$ as shown in Figure 8. [6]

Figure 8

Question 7(c)
Derive an expression for the magnetic field strength at distance d from the centre of a straight wire of finite length L that carries current I. You have to provide a diagram of the straight wire with clear labels of all the quantities you use in your derivation. [8]

Question 8(a)
(i) A wire with resistivity ρ carries current i_C. The current is increasing at the rate di_C/dt. Show that there is a displacement current i_D in the wire equal to $\rho\varepsilon_0\, di_C/dt$. [4]

(ii) Consider a long, straight silver wire with resistivity $\rho = 1.62\times10^{-8}\ \Omega\cdot\text{m}$. The current in the wire is uniform and changing at the rate of $2000\ \text{A/s}$ when the current is 100 A. What is the ratio of the magnitude of the magnetic field due to the displacement current to that due to the current at a small distance r from the wire? [2]

Question 8(b)

The square loop in Figure 9 is made of rigid conducting rods, each $3.00\ \text{m}$ in length, with a total series resistance of $10.0\ \Omega$. It is placed in a uniform $0.100\ \text{T}$ magnetic field directed perpendicularly into the plane of the paper.

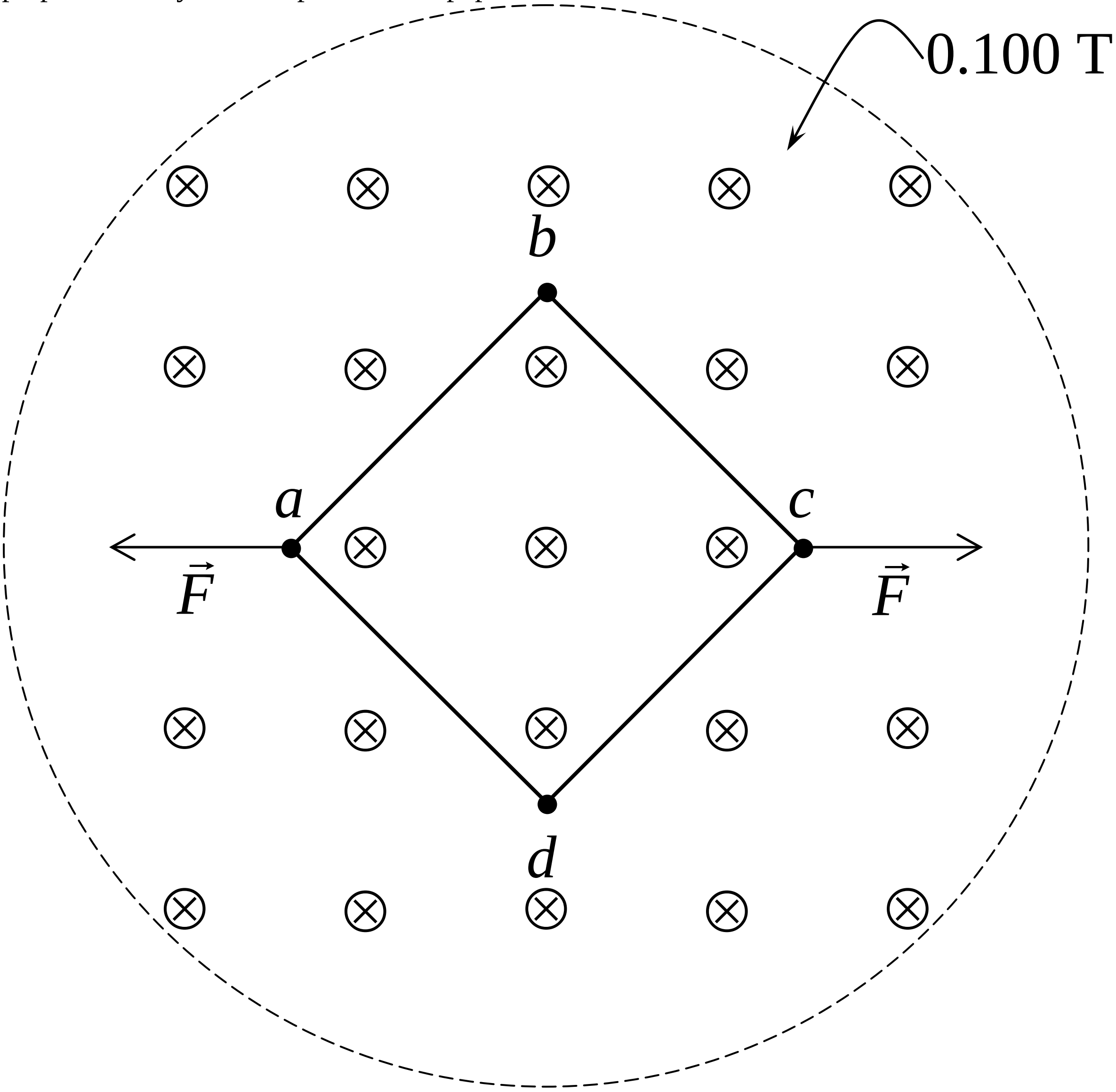

Figure 9

The loop, which is hinged at each corner, is pulled until the separation between points *b* and *d* is $3.00\ \text{m}$ as shown in Figure 10.

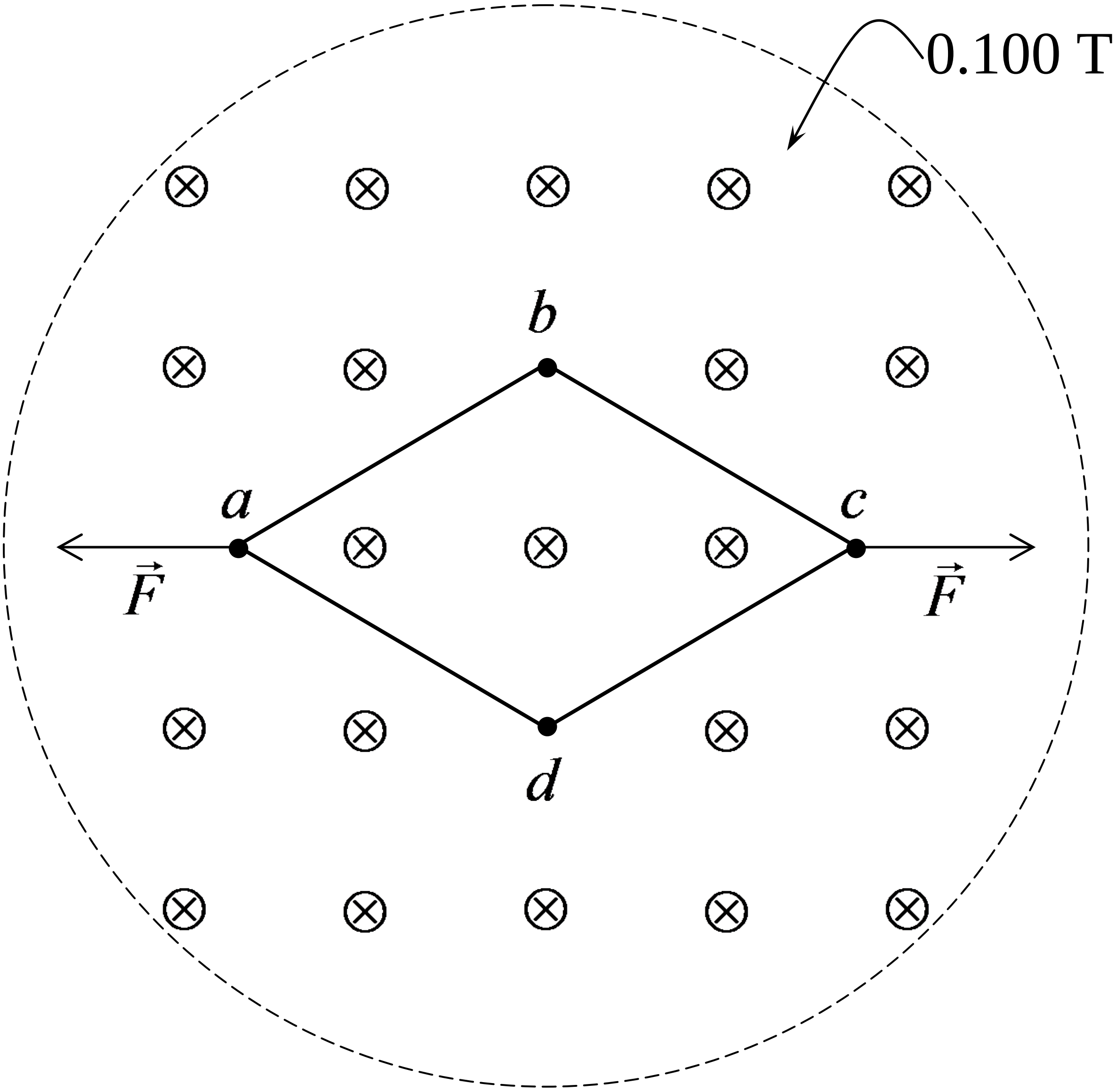

Figure 10

This process takes $0.100\,\mathrm{s}$ to be completed, and is executed in such a way that the rate of change in area bounded by *abcd* is a constant.

(i) What is the direction of the induced current in the loop? Justify your answer. [2]
(ii) What is the power delivered to the resistors? [4]
(iii)Using

$$\mathcal{E}_{ind} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l},$$

show that the induced emf $\mathcal{E}_{ind}$ in the loop at any instant during the 0.100 s is given by

$$\mathcal{E}_{ind} = -1.80 \cos 2\theta \frac{d\theta}{dt},$$

where θ is the angle that *bc* makes with the horizontal through *c*. [8]

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END OF PART II
END OF PAPER