NATIONAL UNIVERSITY OF SINGAPORE

Advanced Placement Test

PC1142 Introduction to Thermodynamics and Optics

(Sample Paper)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **five short** questions in Part I and **three long** questions in Part II. It comprises **5** printed pages including this one.

2. Answer **ALL** questions.

3. Answers to the questions are to be written in the answer books.

4. This is a **CLOSED BOOK** examination.

5. The total mark for Part I is 40 and that for Part II is 60.

6. Only non-programmable and non-graphing calculators without remote communication function may be used.

7. List of constants and formulae are given on Page 2 and 3.

Thermal physics and kinetic theory:

(i) Kinetic mean free path is given by $\ell = \frac{1}{\sqrt{2}\pi \cdot n_v \cdot d^2}$, where n_v is the number density and d is the molecular diameter.

(ii) Maxwell–Boltzmann distribution in 3D:

$$P(v) = 4\pi \cdot v^2 \cdot \left(\frac{m}{2\pi \cdot k_B \cdot T}\right)^{3/2} \cdot \exp\left(-\frac{m \cdot v^2}{2 \cdot k_B \cdot T}\right)$$

(a) Root-mean-square speed: $v_{rms} = \sqrt{\frac{3k_B T}{m}}$.

(b) Average speed: $v_{av} = \sqrt{\frac{8k_B T}{\pi m}}$.

(c) Most-probable speed: $v_{mp} = \sqrt{\frac{2k_B T}{m}}$.

(iii) Ideal gas equation: $p \cdot V = n \cdot R \cdot T$.

(iv) Van der Waals equation:

$$(p + \frac{a \cdot n^2}{V^2}) \cdot (V - n \cdot b) = n \cdot R \cdot T \cdot$$

(v) One form of adiabat: $p \cdot V^\gamma = constant$.

(vi) Stefan–Boltzmann equation: $P_{rad} = \sigma \cdot A \cdot e \cdot T^4$.

(vii) Planck radiation equation:

$$P_{rad}(\lambda) = \frac{2\pi \cdot h \cdot c^2}{\lambda^5 \cdot \left(\exp\left(\frac{h \cdot c}{\lambda \cdot k_B \cdot T}\right) - 1\right)}.$$

(viii) Wien displacement law:

$$\lambda_{peak} = \frac{2.898x10^{-3}\, m \cdot K}{T}.$$

(ix) Convection equation: $P_{conv} = h \cdot A \cdot \Delta T$.

(x) Conduction equation: $P_{ccond} = -k \cdot A \cdot \frac{\Delta T}{L}$.

(xi) Linear thermal expansion: $\Delta L = \alpha \cdot L \cdot \Delta T$.

Thermodynamics:

(i) First law of thermodynamics: $\Delta U = q_{in} + W_{in}$. Gas expansion work done by the gas: $W_{out} = \int p \cdot dV$.

(ii) Carnot heat-engine efficiency $e_C = 1 - \frac{T_C}{T_H}$.

(iii) Entropy $dS = \frac{dq_{rev}}{T}$.

(iv) Enthalpy $H = E + PV$.

(v) Helmholtz free energy $F = E - TS$.

(vi) Gibbs free energy $G = H - TS$.

Geometric optics:

(i) p and q are object and image distances measured respectively on opposite sides of the lens or of the refracting surface:

(a) Object–image relation (thin lens): $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$,

where $\frac{1}{f} = (n-1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

(b) Refracting-surface equation: $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$.

(ii) Gullstrand equation:

(a) Effective power: $P_e = P_1 + P_2 - P_1 \cdot P_2 \cdot \frac{d}{n}$.

(b) Front-vertex refracting power:

$$P_f = P_1 + \frac{P_2}{1 - \frac{P_2 \cdot d}{n}}.$$

(c) Back-vertex refracting power:

$$P_b = P_2 + \frac{P_1}{1 - \frac{P_1 \cdot d}{n}}.$$

(iii) Spherical-mirror equation: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, where $\frac{1}{f}=\frac{2}{R}$.

(iv) *F*-number: $\frac{F}{\#}=\frac{f}{D}$.

(v) Numerical aperture: $NA = n \cdot \sin\theta$.

(vi) Snell's law: $n_1 \cdot \sin\theta_1 = n_2 \cdot \sin\theta_2$.

(a) Critical angle: $\sin\theta_c = n_2 / n_1$.

(b) Brewster angle: $\tan\theta_p = n_2 / n_1$.

(vii) Wave relation: $v = f \cdot \lambda$.

(viii) Abbe number: $v = \frac{n_D - 1}{n_F - n_C}$.

Wave optics:

(i) Circular aperture (Airy's disc): first diffraction minimum is at $\sin\theta = \frac{1.22\lambda}{a}$.

(ii) Slit: first diffraction minimum is at $\sin\theta = \frac{\lambda}{a}$.

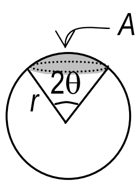
(iii) *N*-slit intensity pattern: $I = I_o \cdot \frac{\sin^2(N\cdot\phi/2)}{\sin^2(\phi/2)}$, where $\phi = \frac{2\pi}{\lambda}\cdot d \cdot \sin\theta$.

(iv) Single-slit diffraction pattern:

$I = I_o \cdot \frac{\sin^2(\delta/2)}{(\delta/2)^2}$, where $\delta = \frac{2\pi}{\lambda}\cdot a \cdot \sin\theta$.

General:

(i) For a circle, the arc length subtended by conical angle α is given by $s = r\,\alpha$.



(ii) For a sphere, the surface area subtended by conical angle 2θ is given by $A = 2\pi r^2 (1 - \cos\theta)$.

(iii) $\log_a (b\,c) = \log_a b + \log_a c$.

(iv) $\log_a b = \log_d b / \log_d a$.

(v) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ except for $n = -1$ for which $\int x^{-1} dx = \ln x + c$.

Universal constants:

Gas constant R = 8.314 J K^{-1} mol^{-1}

Boltzmann constant k_B = 1.381x10^{-23} J K^{-1}

Stefan–Boltzmann constant σ = 5.670x10^{-8} W m^{-2} K^{-4}

Speed of light in vacuum c = 2.998x10^{8} m s^{-1}

Avogadro's number N_A = 6.022x10^{23} mol^{-1}

Permittivity of free space ε_o = 8.854x10^{-12} Fm^{-1}

Planck constant h = 6.626x10^{-34} J s

PC1142 Physics II

Part 1 Answer all FIVE questions. All questions carry 8 marks each.

1. The dark nebula in interstellar space is thought to contain 50 hydrogen (H) atoms per cubic centimetre having a root-mean-square speed of 700 m s^{-1}. [*Hint*: H has an atomic weight of 1.0 g mol^{-1} and a diameter of 1.0 Å.]

   (i) Compute the mean free path of these H atoms.

   (ii) Compute their mean temperature.

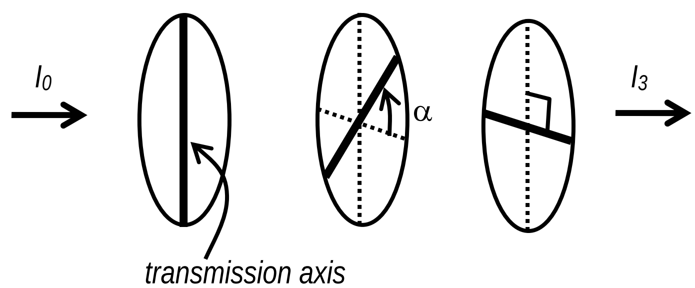
2. The Joule-Thomson coefficient μ_{JT} is the rate of change of temperature of a gas with pressure, $\mu_{JT} = \left(\frac{dT}{dP}\right)_H$, when it is expanded adiabatically against a constant pressure (i.e., at constant enthalpy *H*). According to Euler's chain rule, this in turn is given by $\left(\frac{dT}{dP}\right)_H = -\frac{\left(\frac{dH}{dP}\right)_T}{\left(\frac{dH}{dT}\right)_P}$. Hence or otherwise, deduce the value of μ_{JT} for an ideal gas.

3. At sufficiently low temperatures, the molar heat capacity *C* of non-metals varies according to the Debye T^3 law as $C = k\left(\frac{T}{\theta}\right)^3$, where k = 1940 J mol^{-1} K^{-1}, and θ is a materials property called the Debye temperature. For NaCl, θ = 281 K. Compute the heat per mol required to raise the temperature of NaCl from 1 K to 30 K.

4. The Arecibo radio telescope in Puerto Rico has a 305-m-diameter collector dish and uses radio waves of wavelength 75 cm, instead of visible light, to form images. Compute the diffraction-limited feature size that that can be resolved according to the Rayleigh criteria

on the moon at a distance of 363,000 km. Deduce whether it would be able to resolve a family-size car roaming on the surface of the moon.

5. Unpolarised light of intensity I_o is incident on a stack of three polarisers. The transmission axis of the third polariser is always perpendicular to the first polariser, while that of the second polariser is oriented at an angle α with respect to this third polariser (see diagram below). Derive an expression for the intensity of light I_3 transmitted through the third polariser in terms of α and I_o.



Part 2 Answer all THREE questions. All questions carry 20 marks each.

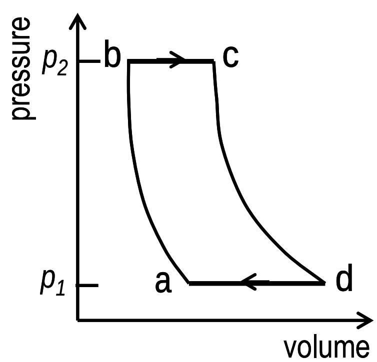
6. The Brayton cycle is often used to describe the operation of gas turbine engines. It comprises the following steps, as shown below:

$a \rightarrow b$: adiabatic compression of air, initially at atmospheric pressure p_1, by a rotary compressor to pressure p_2;

$b \rightarrow c$: isobaric heating at pressure p_2 due to combustion of fuel injected into the hot air;

$c \rightarrow d$: adiabatic expansion of the hot gases through the rotating turbine blades to atmospheric pressure p_1;

$d \rightarrow a$: isobaric cooling at pressure p_1 due to heat rejection into atmosphere.



(i) Sketch the temperature vs entropy state diagram. Indicate on that diagram the locations of *a*, *b*, *c* and *d*, and the direction of the cycle.

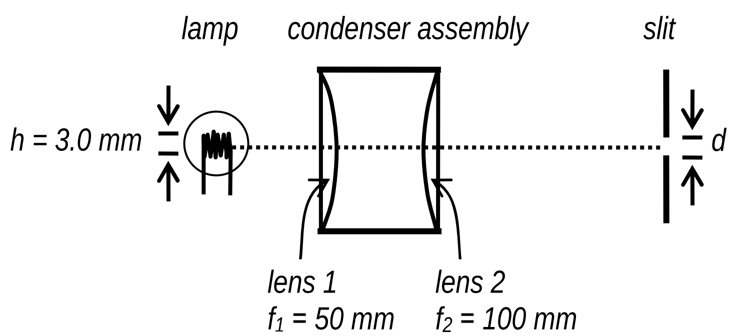
(ii) Derive an expression for the thermal efficiency of this cycle in terms of the pressure ratio p_2/p_1 and the heat capacity ratio of the working gas γ.

7. Light from a tungsten filament lamp is focussed by a condenser assembly onto a slit which is the entrance to an optical instrument. The height of the filament *h* is 3.0 mm (see diagram below). The condenser assembly comprises two fused-silica plano-convex lenses of focal lengths 50 mm and 100 mm, and identical diameters of 50 mm, oriented as shown below and aligned to the optical axis defined by the centres of the filament and of the slit. The centre of the filament is at the focal point (*ie*., 50 mm to the left) of lens 1. The refractive index of fused silica is 1.45.

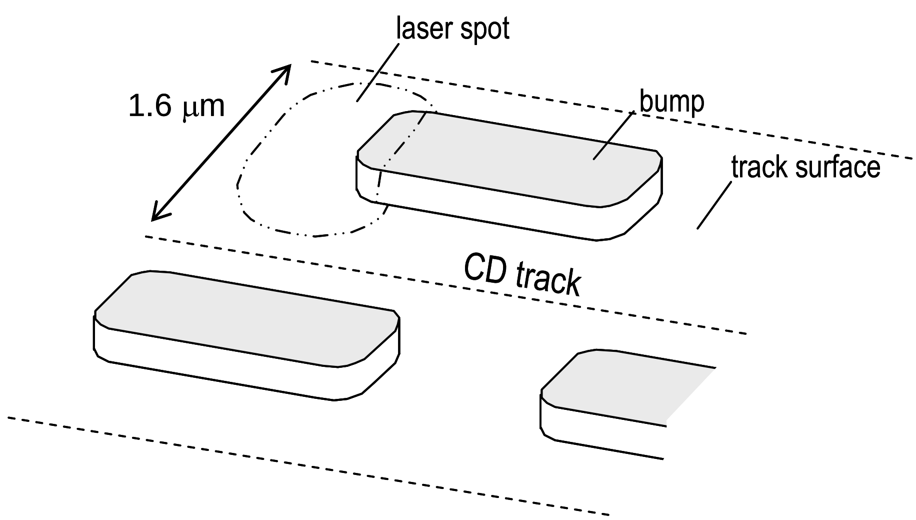
(i) Compute the *F*-number and the radius of curvature of lens 2.

(ii) Compute the location of the image and the minimum slit width *d* required to collect the entire image.

(iii) Explain briefly why it is more desirable to use fused silica lenses than soda lime glass lenses.



8. The information in a compact disc (CD) is encoded as 0.8-μm-wide bumps on a 1.6-μm-wide spiral track on a reflective metallised surface. The spiral puts subsequent segments of the track parallel to each other (schematic shown below). A semiconductor laser with a vacuum wavelength of 0.79-μm is focussed to a spot (also shown) within this track. By spinning the CD, the laser spot is made to scan along the track. As it passes over a bump, roughly half of the spot is reflected from the track surface, and the remainder from the top of the bump. The height of these bumps is chosen to give destructive interference between these two reflections so that the reflected intensity is minimised. The bump and track surface are covered by a millimetre-thick polycarbonate layer with a refractive index *n* of 1.8, through which the laser passes before reflection.



(i) Compute the wavelength and frequency of the laser radiation inside the polycarbonate layer. Write down whether this laser is visible to the human eye.

(ii) Deduce the minimum height of the bump required for the first destructive interference of the reflected laser light.

(iii) Compute the first-order diffraction angle of green light in air due to the periodicity perpendicular to the track direction when the CD surface is illuminated by white light at normal. [*Hint*: The vacuum wavelength of green light is 550 nm.]

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