NATIONAL UNIVERSITY OF SINGAPORE

Advanced Placement Test

PC1141 Introduction to Classical Mechanics

(Sample Paper)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

- 1. This examination paper contains <u>five</u> short questions in Part I and <u>three</u> long questions in Part II. It comprises <u>5</u> printed pages including this one.
- 2. Answer <u>ALL</u> questions.
- 3. Answers to the questions are to be written in the answer books.
- 4. This is a **CLOSED BOOK** examination.
- 5. The total mark for Part I is 40 and that for Part II is 60.
- 6. Only non-programmable and non-graphing calculators without remote communication function may be used.
- 7. List of constants and mathematical formulas are given on Page 2.

List of Constants and Mathematical Formulas

$$G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2$$

$$g = 9.80 \text{m/s}^2$$

$$R_{\text{Earth}} = 6.37 \times 10^6 \text{m}$$

$$M_{\text{Earth}} = 5.97 \times 10^{24} \text{kg}$$

$$(1 + x)^{\alpha} \approx 1 + \alpha x, \text{ for } |x| \ll 1$$

$$\sin(\theta + \delta) \approx \sin \theta + \delta \cos \theta, \text{ for } |\delta| \ll 1$$

Type equation here.

PART I

PART I

This part of the examination paper contains five short-answer questions on page 3. Answer ALL questions. The mark for each question is 8.

- 1. A satellite is in a circular equatorial orbit around the Earth (over the equator). If the satellite flies over Singapore 9 times a day, what is the radius of the orbit?
- 2. A ball is rolling down from the top of a rough spherical dome with negligible initial velocity and angular velocity. Show that the ball must slide before losing the contact with the dome.
- 3. The flattening of the Earth is defined as $f_{\oplus} \equiv 1 R_p / R_e$, where R_e is the radius of the equator and R_p is half of the distance between the North Pole and the South Pole. Estimate the flattening of the Earth.
- 4. A ladder on the rough floor is leaning against a vertical rough wall. The ladder has length *l* and mass *m*. The coefficients of friction are μ for both contact surfaces. What is the smallest angle between the ladder and the floor?
- 5. A runner is running with speed V along the straight line connecting two identical speakers. Both speakers are playing a tone of the same frequency f.
 - (a) What is the beat frequency that the runner hears?
 - (b) If the standing wave forms between two speakers, how frequently will the runner hit a node of the standing wave?
 - (c) Compare your results from (a) and (b).

PART II

This part of the examination paper contains three long-answer questions from page 4 to 5. Answer ALL questions. The mark for each question is 20.

6. A small block with mass m is moving towards a stationary slope with angle θ and height h, as shown in the following figure. Suppose all surfaces are frictionless.



- (a) If the slope is fixed, what is the minimum initial speed for the block to be able to reach the top of the slope?
- (b) If the slope is fixed and the block is given an initial speed v such that it is able to fly off the top of the slope, find the expression of the horizontal distance between the vertical edge of the slope and the block when it reaches the ground.
- (c) Suppose the slope has mass *M* and it is able to move freely on the ground, what is the minimum initial speed for the block to be able to reach the top of the slope?
- In the Cavendish experiment, the two small balls have mass *m* each and are connected by a light rigid rod with length *L*. The two large balls have mass *M* each and are separated by the same distance *L*. The torsion constant of the torsion wire is *K*.



- (a) Find the period for the torsion pendulum when large balls are absent.
- (b) Put the large balls a small distance away from the small balls, when the system reaches the equilibrium, the rigid rod rotates an angle θ and the distance between the centers of balls is r, as shown in the above figure. Find the expression for the gravitational constant G.
- (c) If the small balls are perturbed with small angle from the equilibrium position in (b), will the oscillation be harmonic? If so, find the expression for the period of the oscillation.

8. A cylindrical container of an incompressible liquid with density ρ rotates with constant angular speed ω about its axis of symmetry, which we take to be the *y*-axis.



- (a) At a given distance r to the y-axis, find the expression for the pressure as a function of d, the depth from the lowest point of the liquid surface, as shown in the above figure.
- (b) At a given depth d, find the expression for the pressure as a function of r, the distance to the y-axis.
- (c) From the results in (a) and (b), can you guess a formula for the pressure at an arbitrary point (r,d)? Use your guessed formula to find the expression of the shape of the liquid surface, h(r).

(WQh)

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