NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

ADVANCED PLACEMENT TEST
(SAMPLE)

MA1505 MATHEMATICS I

MMM-YYYY — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. **Write your name here:** _____
2. This paper contains a total of **FIVE (5)** questions and comprises **TWENTY ONE (21)** printed pages, including this page.
3. This is a **CLOSED BOOK** test. No list of formulas is provided and helpsheets are disallowed.
4. Only non-programmable and non-graphing calculators without remote communication function may be used. However, you should lay out systematically the various steps in your calculations.
5. Candidates must answer **ALL** 5 questions.
6. **Write your solutions in the spaces provided below the questions in this test paper. Submit this test paper at the end of the test period.**

For official use only. Do not write below this line.

Question	1	2	3	4	5
(a)					
(b)					

Question 1 (a) [10 marks]

The ellipse C in the xy-plane is given parametrically as follows:

$$C \;:\; x = 6\cos\theta \quad \text{and} \quad y = 2\sin\theta.$$

In the first quadrant, find the **exact value** of the area of the finite region bounded by C, the y-axis and the line L given by

$$L \;:\; y = \frac{1}{\sqrt{3}}\,x.$$

Answer 1(a)	

(Show your working below and on the next page.)

(More working space for Question 1(a))

Question 1 (b) [10 marks]

A tetrahedron is a solid with surface consisting of four triangles. Find the **exact value** of the volume of the tetrahedron with vertices $O(0, 0, 0)$, $A(8, 0, 0)$, $B(7, 6, 5)$ and $C(2, 3, 4)$.

Answer 1(b)	

(Show your working below and on the next page.)

(More working space for Question 1(b))

Question 2 (a) [10 marks]

Find the **exact value** of the sum

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+2)}.$$

Answer 2(a)	

(Show your working below and on the next page.)

(More working space for Question 2(a))

Question 2 (b) [10 marks]

Let

$$f(x) = \frac{x^2 + 1}{x + 1}$$

and let

$$\sum_{n=0}^{\infty} c_n (x + 3)^n$$

be the Taylor series for f at $x = -3$. Find the **exact value** of $c_0 + c_1 + c_{101}$.

Answer 2(b)	

(Show your working below and on the next page.)

(More working space for Question 2(b))

Question 3 (a) [10 marks]

Find the local maximum points, local minimum points, and saddle points, if any, of the function

$$f(x, y) = xy + (x + y)(120 - x - y).$$

Answer 3(a)	

(Show your working below and on the next page.)

(More working space for Question 3(a))

Question 3 (b) [10 marks]

Find the **exact value** of the double integral

$$\int\int_D \sqrt{|x-y|}dxdy,$$

where D is the rectangular region: $0 \leq x \leq 1$ and $0 \leq y \leq 2$.

Answer 3(b)	

(Show your working below and on the next page.)

(More working space for Question 3(b))

Question 4 (a) [10 marks]

Find the **exact value** of the volume of the solid enclosed laterally by the circular cylinder about z-axis of radius 1, bounded on top by the elliptic paraboloid

$$2x^2 + 4y^2 + z = 18 \ ,$$

and bounded below by the plane $z = 0$.

Answer 4(a)	

(Show your working below and on the next page.)

(More working space for Question 4(a))

Question 4 (b) [10 marks]

Let C be the helix parametrised by

$$\mathbf{r}(t) = (3\cos t, 3\sin t, 4t) \quad \text{for } 0 \leq t \leq 4\pi,$$

and let $f(x, y, z) = x^2 + \frac{1}{16}z$. Find the **exact value** of the line integral

$$\int_C f\,ds.$$

Answer 4(b)	

(Show your working below and on the next page.)

(More working space for Question 4(b))

Question 5 (a) [10 marks]

Find the **exact value** of the surface integral

$$\int\int_S z dS,$$

where S is the surface $z = x^2 + y^2$ with $0 \leq z \leq 1$.

Answer 5(a)	

(Show your working below and on the next page.)

(More working space for Question 5(a))

Question 5 (b) [10 marks]

Use Stokes' Theorem to find the **exact value** of the line integral

$$\oint_C (-yzdx + xzdy + xydz),$$

where C is the curve of intersection of the plane

$$x + y + z = 2$$

and the cylinder

$$x^2 + y^2 = 1,$$

oriented in the counterclockwise sense when viewed from above.

Answer 5(b)	

(Show your working below and on the next page.)

(More working space for Question 5(b))

END OF PAPER