

NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF MATHEMATICS
ADVANCED PLACEMENT TEST
(SAMPLE)

MA1505 MATHEMATICS I

MMM-YYYY — Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. Write your name here: _____
2. This paper contains a total of **FIVE (5)** questions and comprises **TWENTY ONE (21)** printed pages, including this page.
3. This is a **CLOSED BOOK** test. No list of formulas is provided and helpsheets are disallowed.
4. Only non-programmable and non-graphing calculators without remote communication function may be used. However, you should lay out systematically the various steps in your calculations.
5. Candidates must answer **ALL** 5 questions.
6. Write your solutions in the spaces provided below the questions in this test paper. Submit this test paper at the end of the test period.

For official use only. Do not write below this line.

Question	1	2	3	4	5
(a)					
(b)					

Question 1 (a) [10 marks]

The ellipse C in the xy -plane is given parametrically as follows:

$$C : x = 6 \cos \theta \quad \text{and} \quad y = 2 \sin \theta.$$

In the first quadrant, find the **exact value** of the area of the finite region bounded by C , the y -axis and the line L given by

$$L : y = \frac{1}{\sqrt{3}} x.$$

Answer 1(a)	
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(Show your working below and on the next page.)

(More working space for Question 1(a))

Question 1 (b) [10 marks]

A tetrahedron is a solid with surface consisting of four triangles. Find the **exact value** of the volume of the tetrahedron with vertices $O(0, 0, 0)$, $A(8, 0, 0)$, $B(7, 6, 5)$ and $C(2, 3, 4)$.

Answer 1(b)	
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(Show your working below and on the next page.)

(More working space for Question 1(b))

Question 2 (a) [10 marks]Find the **exact value** of the sum

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+2)}.$$

Answer 2(a)	
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(Show your working below and on the next page.)

(More working space for Question 2(a))

Question 2 (b) [10 marks]

Let

$$f(x) = \frac{x^2 + 1}{x + 1}$$

and let

$$\sum_{n=0}^{\infty} c_n (x + 3)^n$$

be the Taylor series for f at $x = -3$. Find the **exact value** of $c_0 + c_1 + c_{101}$.

Answer 2(b)	
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(Show your working below and on the next page.)

(More working space for Question 2(b))

Question 3 (a) [10 marks]

Find the local maximum points, local minimum points, and saddle points, if any, of the function

$$f(x, y) = xy + (x + y)(120 - x - y).$$

Answer 3(a)	
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(Show your working below and on the next page.)

(More working space for Question 3(a))

Question 3 (b) [10 marks]

Find the **exact value** of the double integral

$$\int \int_D \sqrt{|x - y|} dx dy,$$

where D is the rectangular region: $0 \leq x \leq 1$ and $0 \leq y \leq 2$.

Answer 3(b)	
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(Show your working below and on the next page.)

(More working space for Question 3(b))

Question 4 (a) [10 marks]

Find the **exact value** of the volume of the solid enclosed laterally by the circular cylinder about z -axis of radius 1, bounded on top by the elliptic paraboloid

$$2x^2 + 4y^2 + z = 18 ,$$

and bounded below by the plane $z = 0$.

Answer 4(a)	
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(Show your working below and on the next page.)

(More working space for Question 4(a))

Question 4 (b) [10 marks]

Let C be the helix parametrised by

$$\mathbf{r}(t) = (3 \cos t, 3 \sin t, 4t) \quad \text{for } 0 \leq t \leq 4\pi,$$

and let $f(x, y, z) = x^2 + \frac{1}{16}z$. Find the **exact value** of the line integral

$$\int_C f ds.$$

Answer 4(b)	
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(Show your working below and on the next page.)

(More working space for Question 4(b))

Question 5 (a) [10 marks]

Find the **exact value** of the surface integral

$$\int \int_S z dS,$$

where S is the surface $z = x^2 + y^2$ with $0 \leq z \leq 1$.

Answer 5(a)	
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(Show your working below and on the next page.)

(More working space for Question 5(a))

Question 5 (b) [10 marks]

Find the **exact value** of the line integral

$$\oint_C (-yzdx + xzdy + xydz),$$

where C is the curve of intersection of the plane

$$x + y + z = 2$$

and the cylinder

$$x^2 + y^2 = 1,$$

oriented in the counterclockwise sense when viewed from above.

Answer 5(b)	
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(Show your working below and on the next page.)

(More working space for Question 5(b))

END OF PAPER